# Simplex Method

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# Real LP Problems

Real-world LP problems often involve:

- Hundreds or thousands of constraints
- Large quantities of data
- Many products and/or services
- Many time periods
- Numerous decision alternatives
- … and other complications
- Complex problem

# Simplex Method

- The simplex algorithm, which was discovered in1947 by George Dantzig, is a simple, straightforward method for solving linear programming problems.

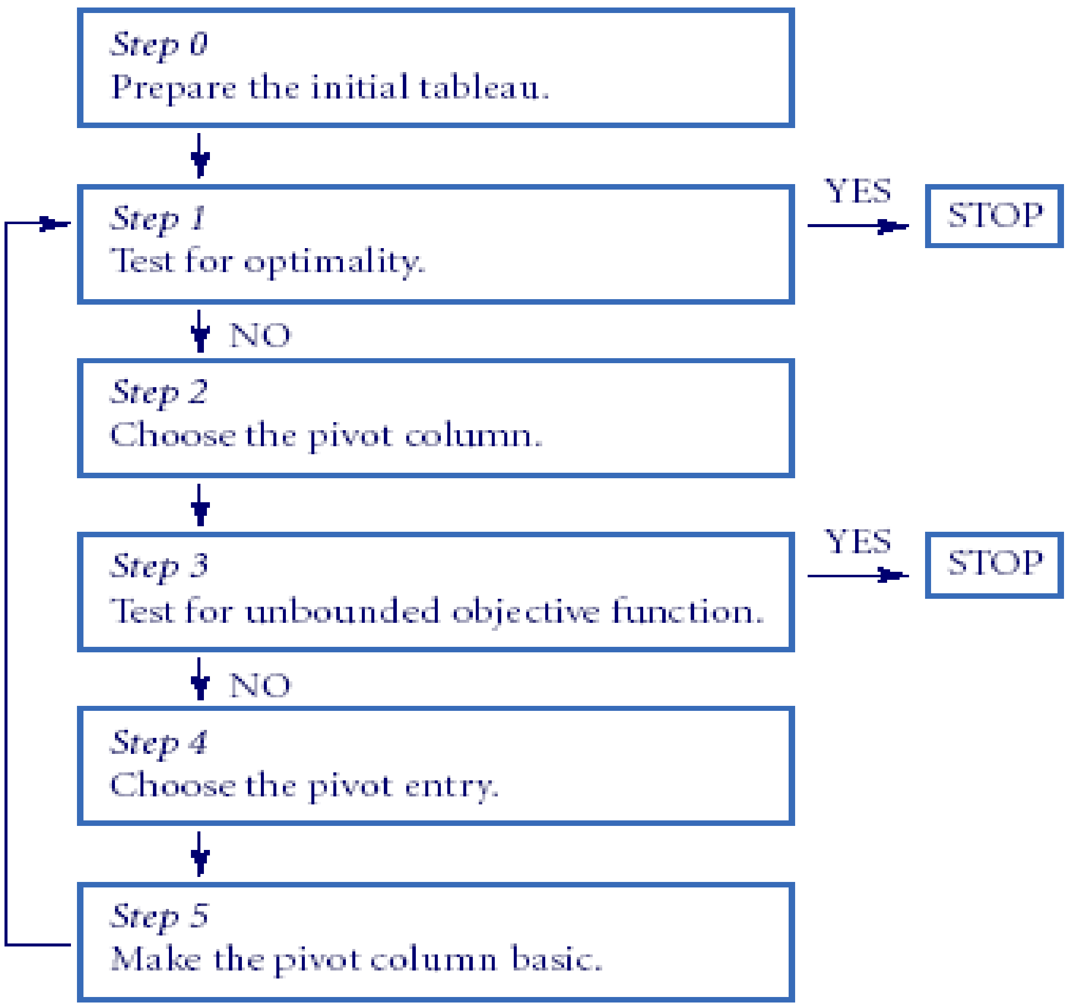
- It has proved to be remarkably efficient method that is used to solve huge problems on today's computers.

# Simplex Method

- Simplex method starts with a feasible solution and tests whether or not it is optimum. If not, the method proceeds a better solution.

- In an algebric procedure, it is much more convenient to deal with equations than with inequality relationships. Therefore, the first step in the setting up the simplex method is to convert inequality constraints into equality constraints. This conversion can be succeeded by introducing ***slack variables***.

# The Simplex Algorithm




All coefficients in the first row are positive

Objective Function has no maximum.

# Simplex Tableaux Formulation

## Problem Statement
### Sleeveless and Sleeve Example

Reebok Sports manufactures two types of t-shirts: sleeveless with logo and sleeve.

**How many sleevelesses and how many slevees should be produced per week, to maximize profits**, given the following constraints...

- **The (profit) contribution per sleeveless is $3.00, compared to $4.50 per sleeve.**
- **Sleeve use 0.5 yards of material; sleeveless use 0.4 yards. 300 yards of material are available.**
- **It requires 1 hour to manufacture one sleeveless and 2 hours for one sleevee. 900 labors hours are available.**
- **There is unlimited demand for sleeveless but total demand for sleeve is 375 units per week.**
- **Each sleeveless uses 1 insignia logo and 600 insignia logos are in stock.**


# LP Formulation

- Maximize $Z=3x_1+4.5x_2$ <u>Objective Function</u>

  where $x_1$=sleeveless, $x_2$=sleeve

| | | | |
|---|---|---|---|
| $0.4x_1+0.5x_2$ | $\leq 300$ | Material | <u>Constraints</u> |
| $x_1+2x_2$ | $\leq 900$ | Labor | |
| $x_2$ | $\leq 375$ | Demand | |
| $x_1$ | $\leq 600$ | Logo | |
| $x_1 \geq 0$, $x_2$ | $\geq 0$ | Nonnegativity | |

# LP Formulation (cont'd)

- Converting inequality constraints into equality constraints by defining *slack variables*.

$$0.4x_1 + 0.5x_2 + x_3 = 300 \quad \text{Material}$$
$$x_1 + 2x_2 + x_4 = 900 \quad \text{Labor}$$
$$x_2 + x_5 = 375 \quad \text{Demand}$$
$$x_1 + x_6 = 600 \quad \text{Logo}$$

- The objective function can be defined as;

$$Z - 3x_1 - 4.5x_2 - 0x_3 - 0x_4 - 0x_5 - 0x_6 = 0$$

instead of

$$Z = 3x_1 + 4.5x_2$$

# 1st Iteration

| Basic variable | Eq. No. | Coefficient of | | | | | | | Right Side |
|---|---|---|---|---|---|---|---|---|---|
| | | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
| Z | 0 | 1 | -3 | -4,5 | 0 | 0 | 0 | 0 | 0 |
| $x_3$ | 1 | 0 | 0,4 | 0,5 | 1 | 0 | 0 | 0 | 300 |
| $x_4$ | 2 | 0 | 1 | 2 | 0 | 1 | 0 | 0 | 900 |
| $x_5$ | 3 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 375 |
| $x_6$ | 4 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 600 |

Pivot column

| |
|---|
| 300/0.5=600 |
| 900/2=450 |
| ✓375/1=375 |

Pivot row

1. Step Determine the *entering basic* variable. Having the largest absolute value in Eq. 0
2. Step Determine the leaving basic variable;
   a) Picking out each coefficient in the column that is strictly positive,
   b) Dividing each of them into “right side” for the same row,
   c) Identifying the the equation that has the smallest ratio.

# GAUSSIAN ELIMINATION

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Row 0 | [ -3 | -4.5 | 0 | 0 | 0 | 0 | : | 0] |
| -(-4.5) | [ 0 | 1 | 0 | 0 | 1 | 0 | : | 375] |
| ------------------------------------------------ | | | | | | | | |
| New Row | [-3 | 0 | 0 | 0 | 4.5 | 0 | : | 1687.5] |

# 2nd Iteration

| Basic variable | Eq. No. | Coefficient of | | | | | | | Basic variable |
|---|---|---|---|---|---|---|---|---|---|
| | | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
| Z | 0 | 1 | -3 | 0 | 0 | 0 | 4,5 | 0 | 1687,5 |
| $x_3$ | 1 | 0 | 0,4 | 0 | 1 | 0 | -0,5 | 0 | 112,5 |
| $x_4$ | 2 | 0 | 1 | 0 | 0 | 1 | -2 | 0 | 150 |
| $x_2$ | 3 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 375 |
| $x_6$ | 4 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 600 |

x5 → x2

| Basic variable | Eq. No. | Coefficient of | | | | | | | Basic variable |
|---|---|---|---|---|---|---|---|---|---|
| | | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
| Z | 0 | 1 | -3 | 0 | 0 | 0 | 4,5 | 0 | 1687,5 |
| $x_3$ | 1 | 0 | 0,4 | 0 | 1 | 0 | -0,5 | 0 | 112,5 |
| $x_4$ | 2 | 0 | 1 | 0 | 0 | 1 | -2 | 0 | 150 |
| $x_2$ | 3 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 375 |
| $x_6$ | 4 | 0 | 1 | 0 | 0 | 0 | 0 | 1 | 600 |

# 3. Iteration

| Basic variable | Eq. No. | Coefficient of | | | | | | | Basic variable |
|---|---|---|---|---|---|---|---|---|---|
| | | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
| Z | 0 | 1 | 0 | 0 | 0 | 3 | -1,5 | 0 | 2137,5 |
| $x_3$ | 1 | 0 | 0 | 0 | 1 | -0,4 | 0,3 | 0 | 52,5 |
| $x_1$ | 2 | 0 | 1 | 0 | 0 | 1 | -2 | 0 | 150 |
| $x_2$ | 3 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 375 |
| $x_6$ | 4 | 0 | 0 | 0 | 0 | -1 | 2 | 1 | 450 |

x4 → x1

| Basic variable | Eq. No. | Coefficient of | | | | | | | Basic variable |
|---|---|---|---|---|---|---|---|---|---|
| | | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
| Z | 0 | 1 | 0 | 0 | 0 | 3 | -1,5 | 0 | 2137,5 |
| $x_3$ | 1 | 0 | 0 | 0 | 1 | -0,4 | 0,3 | 0 | 52,5 |
| $x_1$ | 2 | 0 | 1 | 0 | 0 | 1 | -2 | 0 | 150 |
| $x_2$ | 3 | 0 | 0 | 1 | 0 | 0 | 1 | 0 | 375 |
| $x_6$ | 4 | 0 | 0 | 0 | 0 | -1 | 2 | 1 | 450 |

# Optimal Solution

| Basic variable | Eq. No. | Coefficient of | | | | | | | Basic variable |
|---|---|---|---|---|---|---|---|---|---|
| | | Z | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | |
| Z | 0 | 1 | 0 | 0 | 5 | 1 | 0 | 0 | 2400 |
| $x_5$ | 1 | 0 | 0 | 0 | 10/3 | -4/3 | 1 | 0 | 175 |
| $x_1$ | 2 | 0 | 1 | 0 | 20/3 | 5/3 | 0 | 0 | 500 |
| $x_2$ | 3 | 0 | 0 | 1 | -10/3 | 4/3 | -1 | 0 | 200 |
| $x_6$ | 4 | 0 | 0 | 0 | -20/3 | 5/3 | 0 | 1 | 100 |

- Depending on the optimality test, we found that solution is optimal because none of the coefficients in Eq. 0 are negative, so the algorithm is finished.

- Maximum Profit is $2400 when $x_1$=500 and $x_2$=200.

- Reebok Sports must produce 500 sleeveless and 200 slevees per week to maximize the profit. At this condition the profit will be $2400....!

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