Graphical Methods

Dr. rer.pol. Sudaryanto
sudaryanto@staff.gunadarma.ac.id

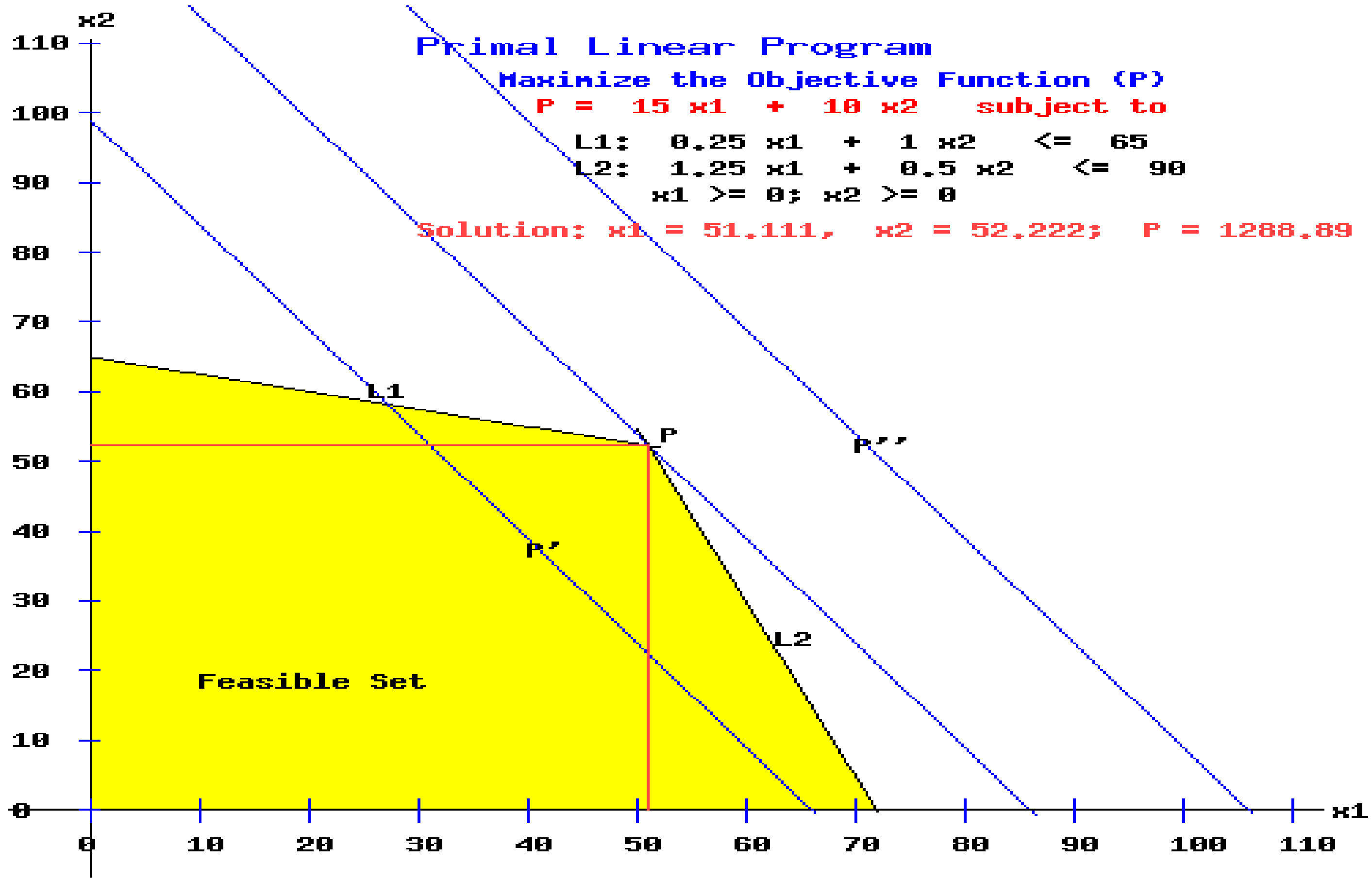
Methods of Solving LP Problems

- Two basic solution approaches of linear programming exist
- The graphical Method
  - simple, but limited to two decision variables
- The simplex method
  - more complex, but solves multiple decision variable problems

Graphical Method

- Construct an x-y coordinate plane/graph
- Plot all constraints on the plane/graph
- Identify the feasible region dictated by the constraints
- Identify the optimum solution by plotting a series of objective functions over the feasible region
- Determine the exact solution values of the decision variables and the objective function at the optimum solution

Graphical Method



LP: Graphical Solution

Solve the following LPP by graphical method

Maximize $Z = 5X_1 + 3X_2$

Subject to constraints

$2X_1 + X_2 \leq 1000$

$X_1 \leq 400$

$X_2 \leq 700$

$X_1, X_2 \geq 0$

Solution: Graphical method

The first constraint $2X_1 + X_2 \leq 1000$ can be represented as follows.

We set $2X_1 + X_2 = 1000$

When $X_1 = 0$ in the above constraint, we get,

$2 \times 0 + X_2 = 1000$

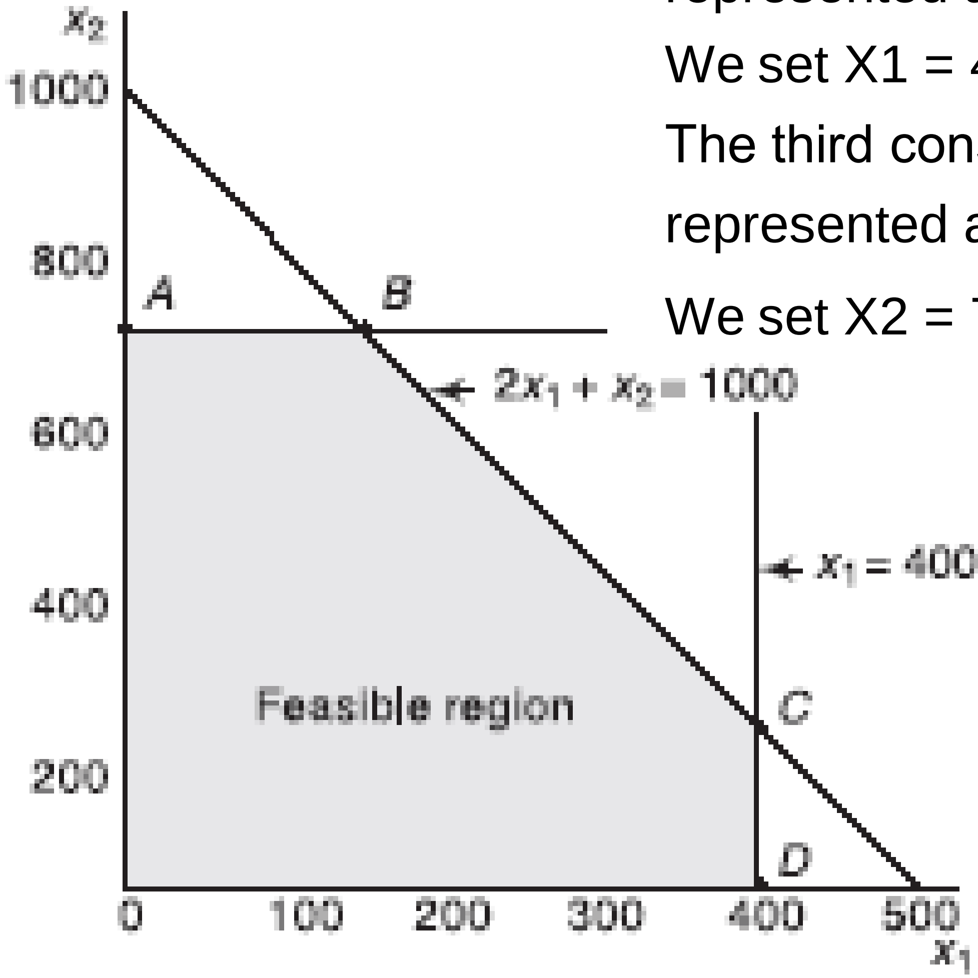
$X_2 = 1000$

Similarly when $X_2 = 0$ in the above constraint, we get,

$2X_1 + 0 = 1000$

$X_1 = 1000/2 = 500$

Solution: Graphical method



The second constraint X1 ≤ 400 can be represented as follows,

We set X1 = 400

The third constraint X2 ≤ 700 can be represented as follows,

We set X2 = 700

LP: Graphical Solution

The constraints are shown plotted in the above figure.

Point	X_1	X_2	$Z = 5X_1 + 3X_2$
0	0	0	0
A	0	700	Z = 5 x 0 + 3 x 700 = 2,100
B	150	700	Z = 5 x 150 + 3 x 700 = 2,850* Maximum
C	400	200	Z = 5 x 400 + 3 x 200 = 2,600
D	400	0	Z = 5 x 400 + 3 x 0 = 2,000

The Maximum profit is at point B When $X_1 = 150$

and $X_2 = 700$, the value 0f Z = <u>2850</u>

Example 2: Graphical Method

Solve the following LPP by graphical method

Maximize $Z = 400X_1 + 200X_2$

Subject to constraints

$18X_1 + 3X_2 \leq 800$

$9X_1 + 4X_2 \leq 600$

$X_2 \leq 150$

$X_1, X_2 \geq 0$

Example 2: Graphical Method

The first constraint $18X_1 + 3X_2 \leq 800$ can be represented as follows.

We set $18X_1 + 3X_2 = 800$

When $X_1 = 0$ in the above constraint, we get,

$18 \times 0 + 3X_2 = 800$

$X_2 = 800/3 = 266.67$

Similarly when X2 = 0 in the above constraint, we get,

18X1 + 3 x 0 = 800

X1 = 800/18 = 44.44

Example 2: Graphical Method

The second constraint $9X_1 + 4X_2 \leq 600$ can be represented as follows,

We set $9X_1 + 4X_2 = 600$

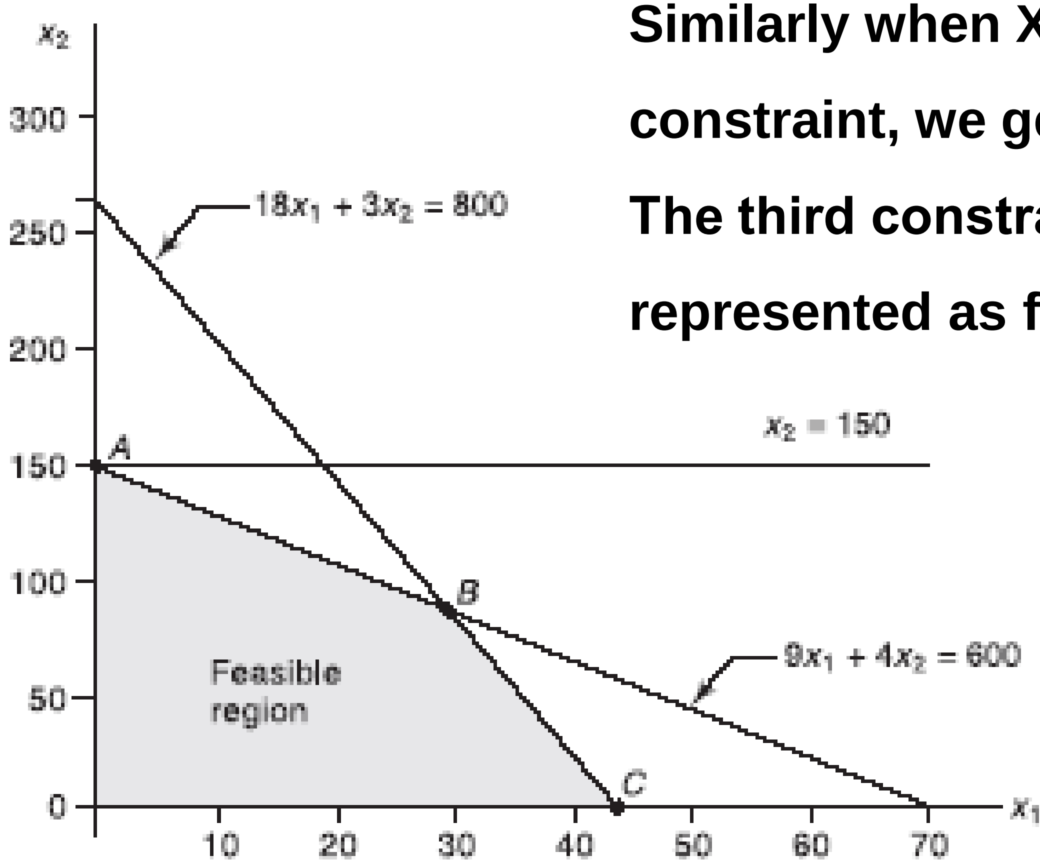
When $X_1 = 0$ in the above constraint, we get,

$9 \times 0 + 4X_2 = 600$

$X_2 = 600/4 = 150$

$X_1 = 600/9 = 66.67$

Example 2: Graphical Method



Similarly when X2 = 0 in the above constraint, we get, 9X1 + 4 x 0 = 600

The third constraint X2 ≤ 150 can be represented as follows, We set X2 = 150

Example 2: The solution

Point	X_1	X_2	$Z = 400X_1 + 200X_2$
0	0	0	0
A	0	150	Z = 400 x 0+ 200 x 150 = 30,000* Maximum
B	31.11	80	Z = 400 x 31.1 + 200 x 80 = 28,444.4
C	44.44	0	Z = 400 x 44.44 + 200 x 0 = 17,777.8

The Maximum profit is at point A When X_1 = 150 and

X_2 = 0 Z = <u>30,000</u>

Example 3: Graphical Method

Solve the following LPP by graphical method

Minimize $Z = 20X_1 + 40X_2$

Subject to constraints

$36X_1 + 6X_2 \geq 108$

$3X_1 + 12X_2 \geq 36$

$20X_1 + 10X_2 \geq 100$

$X_1, X_2 \geq 0$

Example 3: Graphical Method

The first constraint $36X_1 + 6X_2 \geq 108$ can be represented as follows.

We set $36X_1 + 6X_2 = 108$

When $X_1 = 0$ in the above constraint, we get,

$36 \times 0 + 6X_2 = 108$

$X_2 = 108/6 = 18$

Example 3: Graphical Method

Similarly when $X_2 = 0$ in the above constraint, we get,

$36X_1 + 6 \times 0 = 108$

$X_1 = 108/36 = 3$

The second constraint $3X_1 + 12X_2 \geq 36$ can be represented as follows,

We set $3X_1 + 12X_2 = 36$

Example 3: Graphical Method

When $X_1 = 0$ in the above constraint, we get,

$3 \times 0 + 12X_2 = 36$

$X_2 = 36/12 = 3$

Similarly when $X_2 = 0$ in the above constraint, we get,

$3X_1 + 12 \times 0 = 36$

$X_1 = 36/3 = 12$

Example 3: Graphical Method

The third constraint $20X_1 + 10X_2 \geq 100$ can be represented as follows,

We set $20X_1 + 10X_2 = 100$

When $X_1 = 0$ in the above constraint, we get,

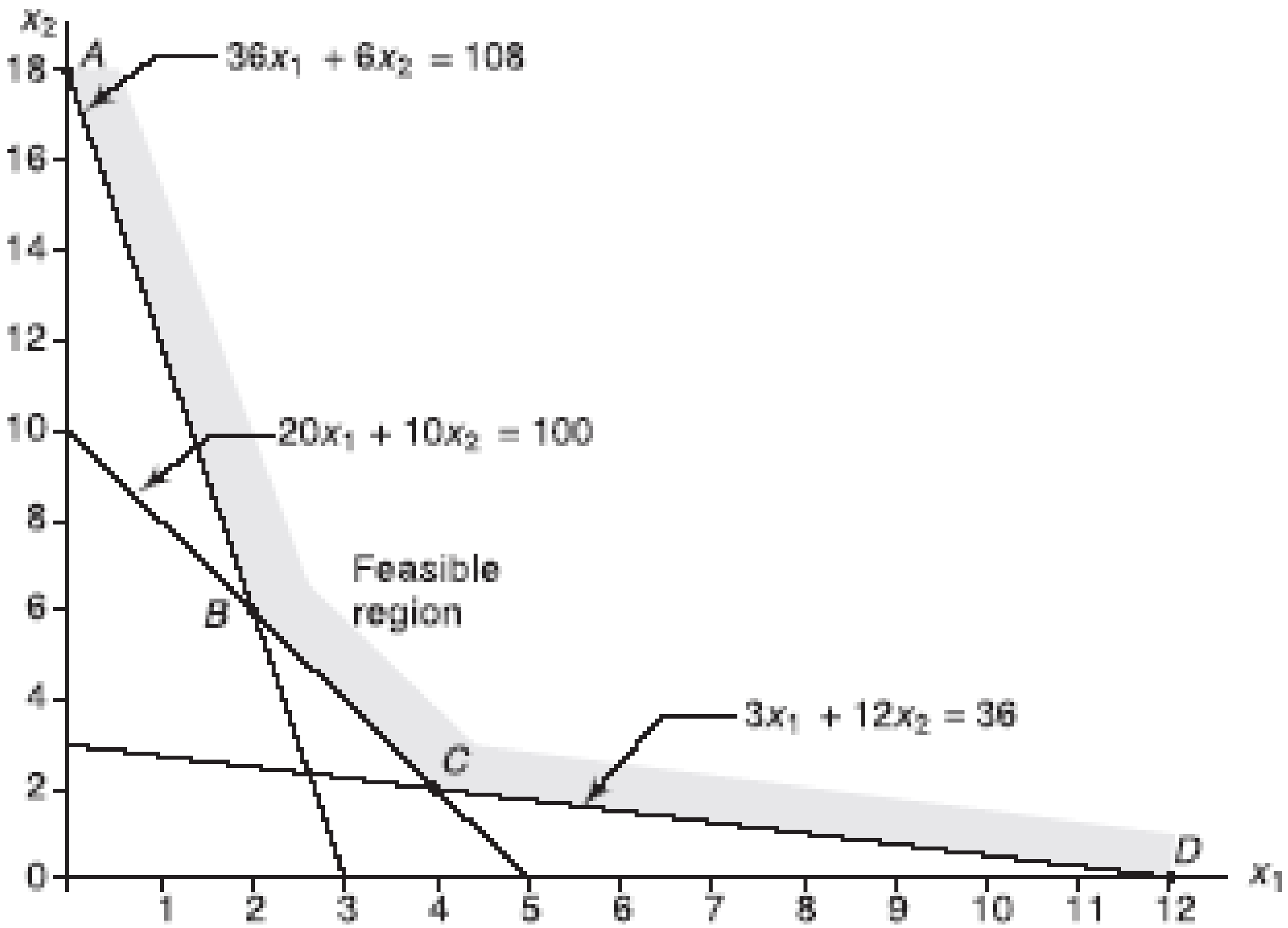
$20 \times 0 + 10X_2 = 100$; $X_2 = 100/10 = 10$

Similarly when $X_2 = 0$ in the above constraint, we get,

$20X_1 + 10 \times 0 = 100$

$X_1 = 100/20 = 5$

Example 3: Graphical Method



Example 3: Graphical Method

Point	X_1	X_2	$Z = 20X_1 + 40X_2$
0	0	0	0
A	0	18	Z = 20 x 0 + 40 x 18 = 720
B	2	6	Z = 20 x2 + 40 x 6 = 280
C	4	2	Z = 20 x 4 + 40 x 2 = 160* Minimum
D	12	0	Z = 20 x 12 + 40 x 0 = 240

The Minimum cost is at point C

When $X_1 = 4$ and $X_2 = 2$; $Z = 160$

Example 4: Graphical Method

Solve the following LPP by graphical method

Maximize $Z = 2.80X_1 + 2.20X_2$

Subject to constraints

$X_1 \leq 20{,}000$

$X_2 \leq 40{,}000$

$0.003X_1 + 0.001X_2 \leq 66$

$X_1 + X_2 \leq 45{,}000$

$X_1, X_2 \geq 0$

Example 4: Graphical Method

The first constraint $X1 \leq 20{,}000$ can be represented as follows.

We set $X1 = 20{,}000$

The second constraint $X2 \leq 40{,}000$ can be represented as follows,

We set $X2 = 40{,}000$

Example 4: Graphical Method

The third constraint $0.003X_1 + 0.001X_2 \leq 66$ can be represented as follows, We set $0.003X_1 + 0.001X_2 = 66$

When $X_1 = 0$ in the above constraint, we get,

$0.003 \times 0 + 0.001X_2 = 66$

$X_2 = 66/0.001 = 66{,}000$

Similarly when $X_2 = 0$ in the above constraint, we get,

$0.003X_1 + 0.001 \times 0 = 66$

$X_1 = 66/0.003 = 22{,}000$

Example 4: Graphical Method

The fourth constraint $X_1 + X_2 \leq 45{,}000$ can be represented as follows, We set $X_1 + X_2 = 45{,}000$

When $X_1 = 0$ in the above constraint, we get,

$0 + X_2 = 45{,}000$

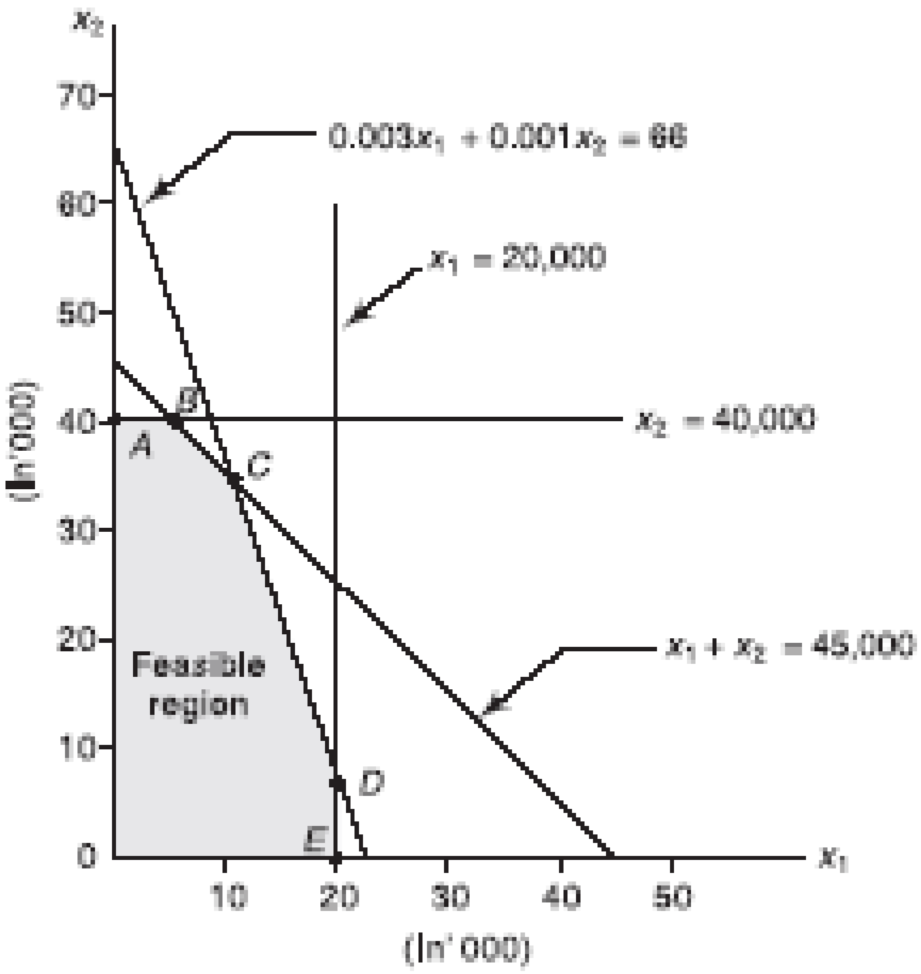
$X_2 = 45{,}000$

Similarly when $X_2 = 0$ in the above constraint, we get,

$X_1 + 0 = 45{,}000$

$X_1 = 45{,}000$

Example 4: Graphical Method



Example 4: Solution

Point	X1	X2	Z = 2.80X1 + 2.20X2
0	0	0	0
A	0	40,000	Z = 2.80 x 0 + 2.20 x 40,000 = 88,000
B	5,000	40,000	Z = 2.80 x 5,000 + 2.20 x 40,000 = 1,02,000
C	10,500	34,500	Z = 2.80 x 10,500 + 2.20 x 34,500 = 1,05,300* Maximum
D	20,000	6,000	Z = 2.80 x 20,000 + 2.20 x 6,000 = 69,200
E	20,000	0	Z = 2.80 x 20,000 + 2.20 x 0 = 56,000

The Maximum profit is at point C When X1 = 10,500 and X2 = 34,500; Z = 1,05,300

Example 5: Graphical Method

Solve the following LPP by graphical method

Maximize $Z = 10X_1 + 8X_2$

Subject to constraints

$2X_1 + X_2 \leq 20$

$X_1 + 3X_2 \leq 30$

$X_1 - 2X_2 \geq -15$

$X_1\ X_2 \geq 0$

Example 5: Graphical Method

The first constraint $2X_1 + X_2 \leq 20$ can be

represented as follows.

We set $2X_1 + X_2 = 20$

When $X_1 = 0$ in the above constraint, we get,

$2 \times 0 + X_2 = 20$

$X_2 = 20$

Example 5: Graphical Method

Similarly when $X_2 = 0$ in the above constraint, we get,

$2X_1 + 0 = 20$

$X_1 = 20/2 = 10$

The second constraint $X_1 + 3X_2 \leq 30$ can be

represented as follows,

We set $X_1 + 3X_2 = 30$

Example 5: Graphical Method

When $X_1 = 0$ in the above constraint, we get,

$0 + 3X_2 = 30$

$X_2 = 30/3 = 10$

Similarly when $X_2 = 0$ in the above constraint, we get,

$X_1 + 3 \times 0 = 30$

$X_1 = 30$

Example 5: Graphical Method

The third constraint $X_1 - 2X_2 \geq -15$ can be represented as follows,

We set $X_1 - 2X_2 = -15$

When $X_1 = 0$ in the above constraint, we get,

$0 - 2X_2 = -15$

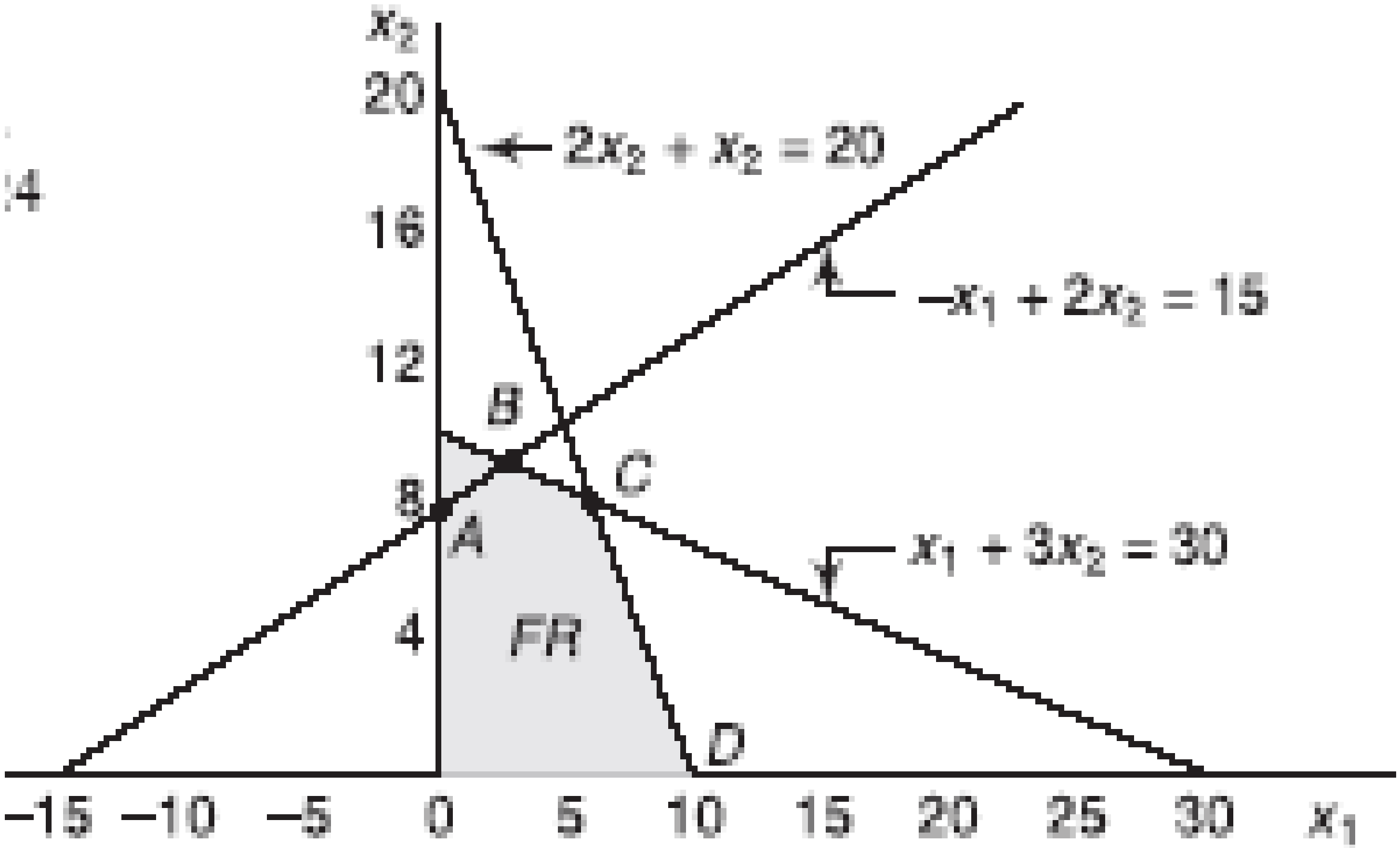
$X_2 = -15/2 = 7.5$

Similarly when $X_2 = 0$ in the above constraint, we get,

$X_1 – 2 \times 0 = -15$

$X_1 = -15$

Example 5: Graphical Method



Example 5: Solution

Point	X_1	X_2	$Z = 10X_1 + 8X_2$
0	0	0	0
A	0	7.5	Z = 10 x 0 + 8 x 7.5 = 60
B	3	9	Z = 10 x 3 + 8 x 9 = 102
C	6	8	Z = 10 x 6 + 8 x 8 = 124* Maximum
D	10	0	Z = 10 x 10 + 8 x 0 = 100

The Maximum profit is at point C, When $X_1 = 6$ and $X_2 = 8$; Z = <u>124</u>

END